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Infrastructure Inspection During and After Unexpected Events - Phase II

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MATC

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Executive Summary

This report presents the development of a finite element (FE) model updating scheme of a highway bridge. In this process, the FE model, based on the original drawing of the bridge, generates a numerical data set that is used to build regression surfaces between the models' parameters and the model's response represented by the natural frequencies and the local transmissibility. Measurements are then conducted on the physical model of the bridge at crucial nonstationary nodes on the superstructure, and the corresponding natural frequencies and local transmissibility are determined from the power spectral density (PSD) and the PSD ratios of the responses, respectively. Both transient as well as stationary excitation methods were exploited to extract system modal properties. An iterative process is then applied to identify the unknown parameters of the physical model by matching the experimental responses with the available parameters in the numerical data set. The proposed method was tested on a highway-bridge model with elastic foundations and showed very promising results.

Chapter 1 Introduction

The development of finite element (FE) models of existing structures is a challenging process, where aging, deterioration, settlement, and corrosion can change the mechanical properties of the physical structure and its boundaries with time [1]. Extensive work has been done for many decades to develop efficient model updating schemes for the FE model. This process involves creating an FE model of the structure as it is built, gathering measurement data from the existing physical structure, and then using the data to update the FE model so that it behaves similarly to the physical structure under different dynamics loadings [2]. It is inevitable that experimental and numerical models deviate from each other, and that can occur for several reasons, such as modeling idealizations, model simplifications, and roundoff errors in numerical models associated with computer-based processes [3]. Structural model updating (SMU) can be categorized as the first essential step for practical structural health monitoring [4]. During the updating process, the numerical model is tuned to match the in situ real-world one, normally by minimizing the discrepancies between their responses.

One major challenge in the FE model updating implementation is that the process requires numerous degrees of freedom (DOFs) to be measured from the physical model in order to determine the appropriate modal parameters/features [5]. In practice, however, it is desired to use a minimum number of sensors to extract the features during vibration-based analysis [6]. A minimum number of features is normally needed in the model updating process as higher modes usually attain less accuracy [7]. Knowing the required number of features can make the model updating process less challenging and more efficient. The conventional optimal sensor installation is normally performed based on stochastic methods [8, 9]. An extensive review of the prevalent optimal sensor placement was done by Mallardo and Aliabadi [10]. This report

presents an approach for model updating of a highway bridge model and its Boundary Conditions (BCs) using a minimum number of measurement points. The proposed method uses a visual inspection toolbox to inspect the uniqueness of the resulting parameters.

Chapter 2 Finite Element Model Updating

2.1 Conventional Structural Model Updating

The conventional model updating schemes are normally based on minimizing a scalar-valued-weighted-sum function, which is constructed to measure a distance between the analytical and experimental modal properties. The following is an example [11-13]:

$$\min : J(\underline{X}) = \sum_{i=1}^N \omega_i^f \left(\frac{f_{a,i} - f_{e,i}}{f_{e,i}} \right)^2 + \sum_{i=1}^N \omega_i^\varphi \left(\frac{1 - MAC_i}{MAC_i} \right)^2 \quad (2.1)$$

$$S.T.: LB \leq \underline{X} \leq UB \quad (2.2)$$

$$\sum_{i=1}^N (\omega_i^f + \omega_i^\varphi) = 1 \quad (2.3)$$

where $\underline{X} = \{x_1 \dots x_M\}$, is a vector that contains unknown system parameters defined as random variables (RV). N and M are the total number of modes and total number of unknown system parameters, respectively. LB and UB are the lower and upper bounds, respectively. The feasible region (FR) of the unknown parameters is defined by the latter bounds. MAC is the modal assurance criterion, which measures the correlation between a pair of local mode shapes [14, 3]. Lastly, $f_{a,i}$, $f_{e,i}$, ω_i^f and ω_i^φ are the i th analytical natural frequency (NF), experimental NF, NF weighting coefficient, and mode shape coefficient of the i th mode.

2.2 Proposed Structural Model Updating Methodology

The SMU methodology aims to utilize a minimum number of essential global and local modal properties using a minimum number of measurement points. A flowchart of the proposed methodology is presented in figure 2.1.

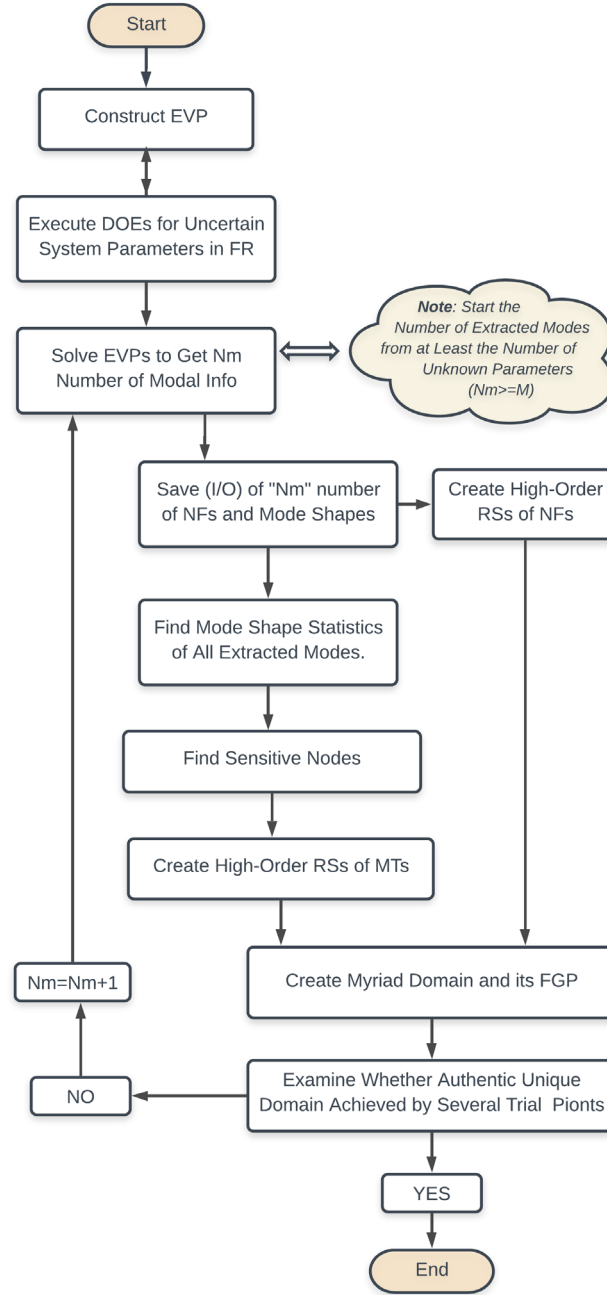


Figure 2.1 Flow chart of the proposed structural model updating process.

As shown in figure 2.1, the first step in the process is to *solve an eigenvalue problem (EVP)* using a detailed finite element model (FEM) of the structure. The second step is to *establish implicit input-output (I/O) relationships* between system parameters (i.e. RV) and

system features. In this process, design of experiments (DOE) must be used to generate a few random points inside the FR [15]. Those random points are combinations of the system's RVs stochastically distributed within their FR [16]. The third stage *saves the EVP*. In this step, the system's uncertain parameters, $\underline{X} = \{x_1 \dots x_M\}$, construct a vector of M number of RVs. Each system feature y_n , is either an eigenvalue or an eigenfunction. The EVP solver returns the system's features, $\underline{Y} = \{f_i, \varphi_i: i = 1: N\}$, and this is constructed by $2N$ of eigen solutions.

The fourth step is to *determine the sensitive nodes*. While global structural features (i.e. NFs) can be extracted from any FEM nodes except modal nodes, the local modal properties (MTi) are highly dependent on the sensor mounting locations. The major goal of this step is to locate sensitive nodes/DOFs where mode shapes oscillate the most when RVs change in FR. It was found that the most appropriate locations for sensor mounting are nodal points that attain high standard deviation (STD).

The fifth step *creates global and local modal properties response surfaces (RSs)*. Since the FEM is kind of a black box and the interrelationship of various I/O parameters are unknown, the RS method is presented in this work to substitute the FEM by mathematical expressions [16, 17]. The general high-order polynomial RS model plus its interaction terms with the system feature (y) can be expressed as [18, 19]:

$$y(x_1 \dots x_M) = \sum_{i_1=0}^p \sum_{i_2=0}^{p-i_1} \sum_{i_3=0}^{p-i_1-i_2} \dots \sum_{i_M=0}^{p-i_1-i_2-i_3 \dots i_{M-1}} \beta_{i_1 i_2 \dots i_M} x_1^{i_1} x_2^{i_2} x_3^{i_3} \dots x_M^{i_M} + \varepsilon \quad (2.4)$$

where $\beta_{i_1 i_2 \dots i_M}$, x_n , ε , M , and p are the regression coefficients, n th independent variable or predictor parameter, Gaussian error term, total number of independent variables, and highest order of polynomials, respectively. The major indicator that shows the goodness of the curve-

fitted surfaces/functions is the coefficient of determination R^2 . This indicator represents the ratio of the sum of the square regression (SSR) to the total sum of the square (SST) as follows [1]:

$$R^2 = \frac{SSR}{SST} = \frac{\sum_{i=1}^n (\mathcal{Y}_i - \bar{y})^2}{\sum_{i=1}^n (\mathcal{Y}_i - \bar{y})^2} \quad (2.5)$$

This coefficient (R^2) can vary from 0.0, which indicates no accuracy, to 1.0. Usually, the RSs are being estimated as second-order ($p = 2$, quadratic) or at most third-order ($p = 3$, cubic) polynomials [4, 20]. It has been proven that the number of regression coefficients required for precise curve fitting grows exponentially as the polynomial degree gets larger [21, 22]. So, the order of the polynomials was gradually increased until high accuracy ($R^2 \geq 0.99$) was attained.

In the sixth step, the *myriad discrete set is created*. In this process, the infinite continuous multidimensional space of FR is discretized to a myriad number (N_D) of RVs that covers FR using the same approach as the second step. Since the parallel coordinate can only plot a discrete set of multidimensional points, this step must be executed for further visualization.

The seventh step *comprises the I/O evaluation of the myriad set from RSs*. In this process, the myriad set of RVs must be evaluated by those RSs estimated from the global and local modal properties in the fifth step. This process is computationally trivial since there is no longer a need to run FEM code; instead, the RSs can be evaluated for the myriad RVs. At the end of this step, the I/O values of the myriad points can be estimated in a meta data format $\{\underline{X}; \underline{Y}\}$. Because each feature has its own variability as RVs change inside the FR, each feature

receives its own weight, which is the specific percent of its own range/variability. That percent is equal to the feature gap percent (FGP). In the constructed set $\{\underline{\mathbf{X}}: \underline{\mathbf{Y}}\}$, FGP is the maximum difference between two successive, or sequentially ordered, features in the myriad set, each feature divided by its own range as follows:

$$FGP = \text{Max}\{ \text{Distance}[\text{Ordered}(y_i)] / \text{Range}(y_i) ; i = 1:N \} \quad (2.6)$$

The final step *determines the minimum required modal information*. This vital stage indicates the minimum number of modal properties Nm that can provide enough constraints to find M number of system RVs in the unique domain (UD). The UD means that, although there is usually more than one unique solution during the modal updating process, those returned optimum points of RVs reside around a single neighborhood inside the FR. The iterative loop was defined to reach a UD from the smallest number of modes $Nm = 1$, as shown in figure 2.1. In each iteration, a trial point of system parameters $\underline{\mathbf{X}}(\mathbf{i})$ was randomly generated within FR in the same manner as in the second stage. Then the system features $\underline{\mathbf{Y}}(\mathbf{i})$ that contain Nm modal information can be estimated in a similar way as in the third stage. Now, even if the exact value of $\underline{\mathbf{Y}}(\mathbf{i})$ does not exist in the constructed discrete myriad set $\{\underline{\mathbf{X}}: \underline{\mathbf{Y}}\}$, the algorithm will still be able to find optimum system parameters $\underline{\mathbf{X}}_o$ in the set as follows:

$$\text{Optimum Points} = \text{Find} \{ \underline{\mathbf{X}}_o | \underline{\mathbf{Y}}(\mathbf{i}) - \underline{\mathbf{B}}_Y \leq \underline{\mathbf{Y}} \leq \underline{\mathbf{Y}}(\mathbf{i}) + \underline{\mathbf{B}}_Y \} \quad (2.7)$$

where feature bound vector ($\underline{\mathbf{B}}_Y$) is defined to provide a fine interval by multiplying the FGP to the distinct feature range in FR as follows:

$$\underline{B_Y} = FGP * Range\langle \underline{Y} \rangle \tag{2.8}$$

Chapter 3 Experimental Implementation and Verification

In this work, Matlab® was adopted for DOE, high-order polynomial curve-fitting, and parallel coordinate visualization. Abaqus® and the Abaqus2Matlab toolbox [23] were exploited for the eigen solver machine and data collection purposes.

3.1 Highway Bridge Model

A small-scale model of a highway bridge superstructure was replicated from in-built drawings received from the Iowa Department of Transportation. The prototype is the superstructure of the westbound US-30 bridge over the Cedar River located southeast of Cedar Rapids, Iowa (FHWA # 33472). The goal was to develop a small-scale model that would behave in a manner similar to the prototype bridge. LEXAN™ 9034 polycarbonate material was used to construct the bridge model girders and deck. In order to satisfy the similarity laws, some concentrated masses were distributed alongside the model. The model consists of girders, cross beams, a deck, and concentrated masses, plus left and right rotational springs as shown in figure 3.1(a). All added masses, the attached shaker, and all other steel bolts and nuts were simulated in the FEM code as concentrated masses. The four potential RVs are $\{E, \rho, K_{\theta l}, K_{\theta r}\}$, representing the Young's modulus of elasticity of the plastic plate used in the deck and girders of the model, the density of the plastic material, and the left and right sides of the rotational stiffness of the bearing system, respectively. The first six mode shapes of the FEM model are shown in figure 3.1(b), while those RVs are kept at their nominal values.

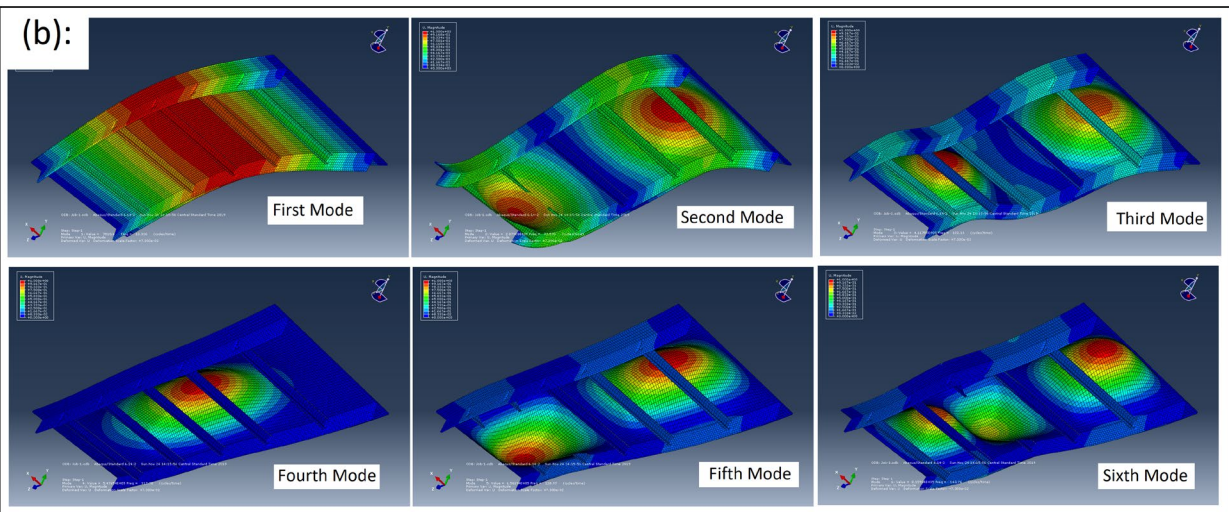
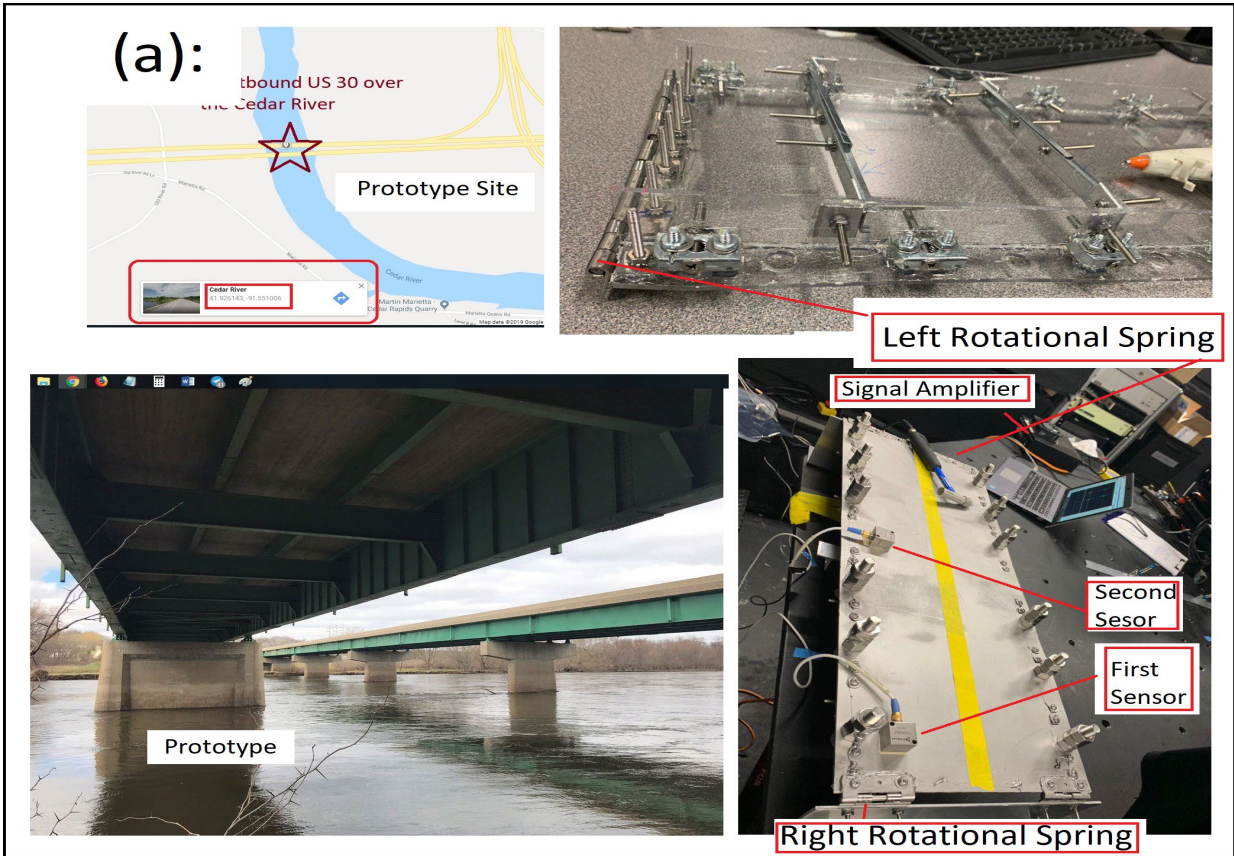


Figure 3.1 (a) The targeted bridge model defined as a prototype, small-scale model plus mounted sensors, electrodynamic shaker, Data Acquisition System (DAS) in experimental configuration; (b) the first six modes of the beam FEM while its RVs are assigned based on the nominal values.

The overlaid mode shapes of the six initial modes along their nodal STD are plotted in figure 3.2. Two high STD nodes were identified for sensor mounting purposes. The highest polynomial order ($p = 5$), the myriad set size ($N_D = 4E6$), and the smallest number of modal properties at sensitive nodes to find UD authentically ($N_m = 4$) were diagnosed by the algorithm.

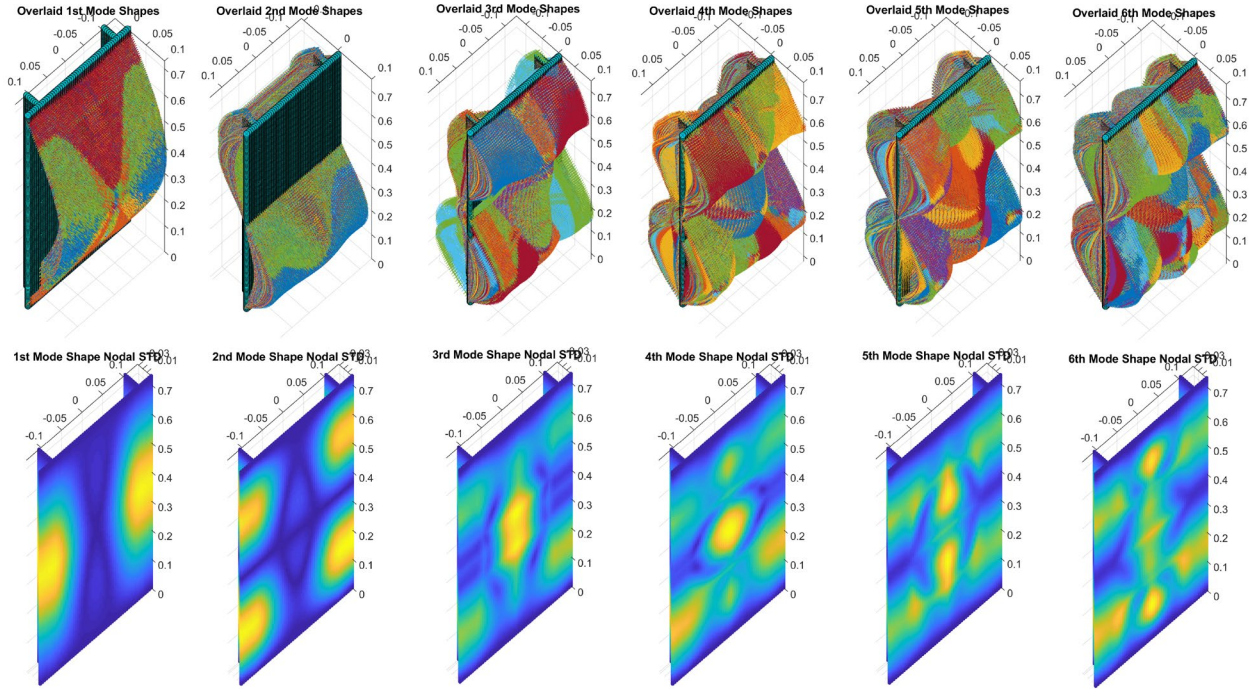


Figure 3.2 The 350 overlaid mode shapes and their standard deviations for the six initial mode shapes.

While the RSs cannot be plotted, since four independent variables exist, the statistical information for the global and local features of the 350 samples are presented in figure 3.3 to check their sensitivity. Figure 3.4 shows the plots of the essential signal processing to execute steady-state incitation performed by a shaker with those three pseudo-random signals. In figure 3.5, a parallel coordinate plot of the myriad set as well as a gradual converging to the optimum

points from two modal properties up to four modes are shown. For the purpose of parallel coordinate visualization, since RVs have different units and ranges, each system parameter in the myriad set was standardized by $(Z_i = X_i - \mu_{Xi}/\sigma_{Xi})$ and then plotted in figure 3.5.

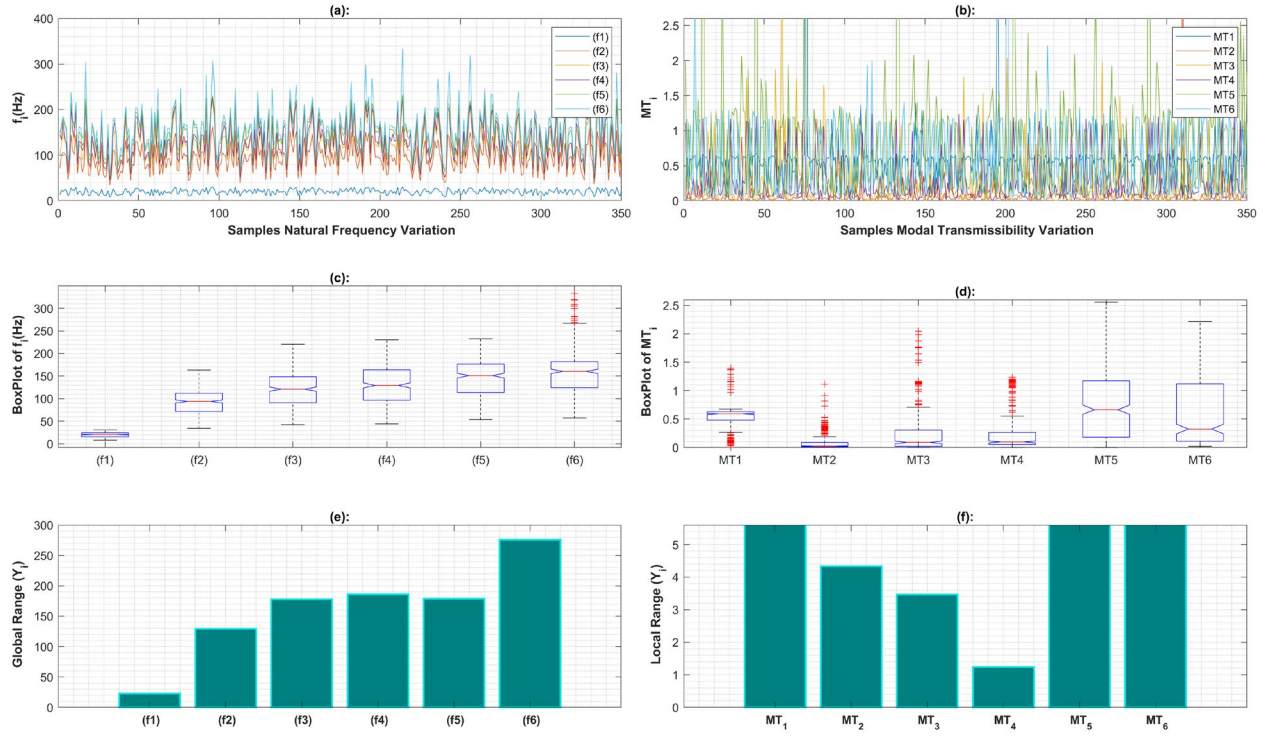


Figure 3.3 The 350 sample results of the bridge model: (a) NFs variation; (b) modal transmissibility (MT) variations; (c) global information statistics; (d) local information statistics; (e) global feature bounds; (f) local feature bounds.

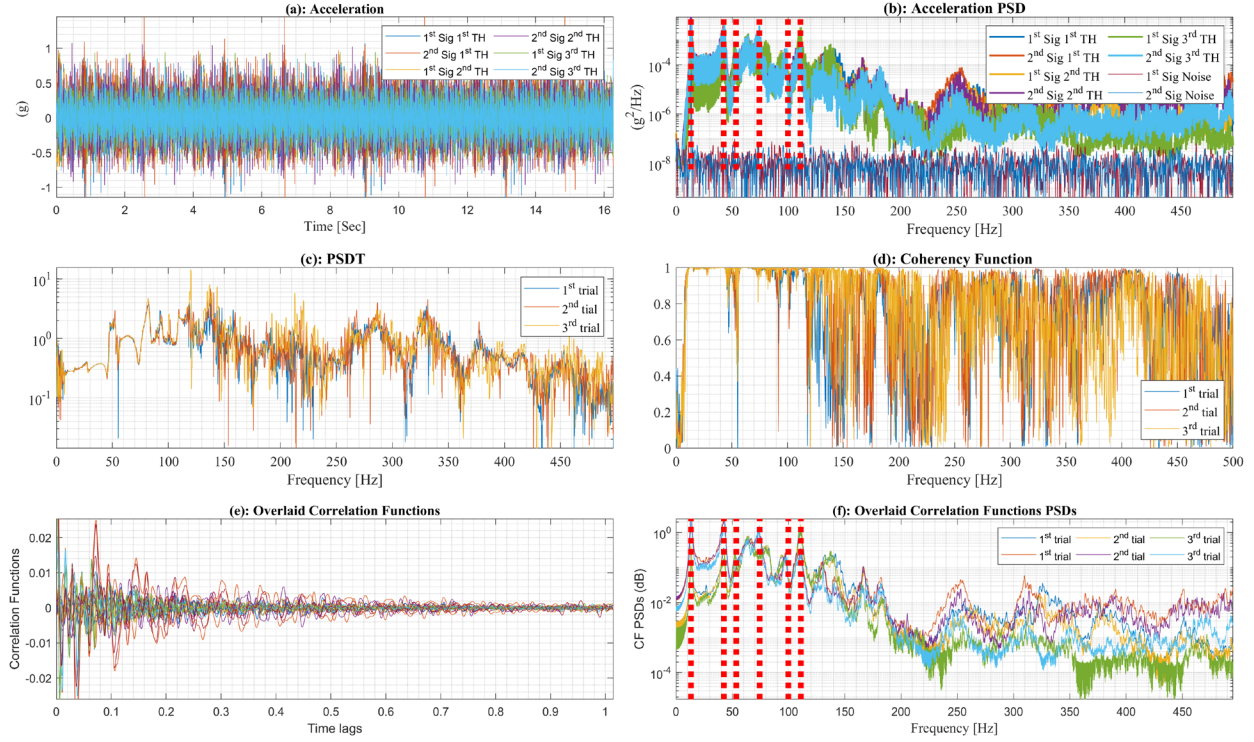


Figure 3.4 The signal processing of the bridge: (a) acceleration signals, (b) PSDs of responses, (c) PSD Transmissibility (PSDTs), (d) coherence function (CFs), (e) overlaid CFs, (f) CF PSDs.

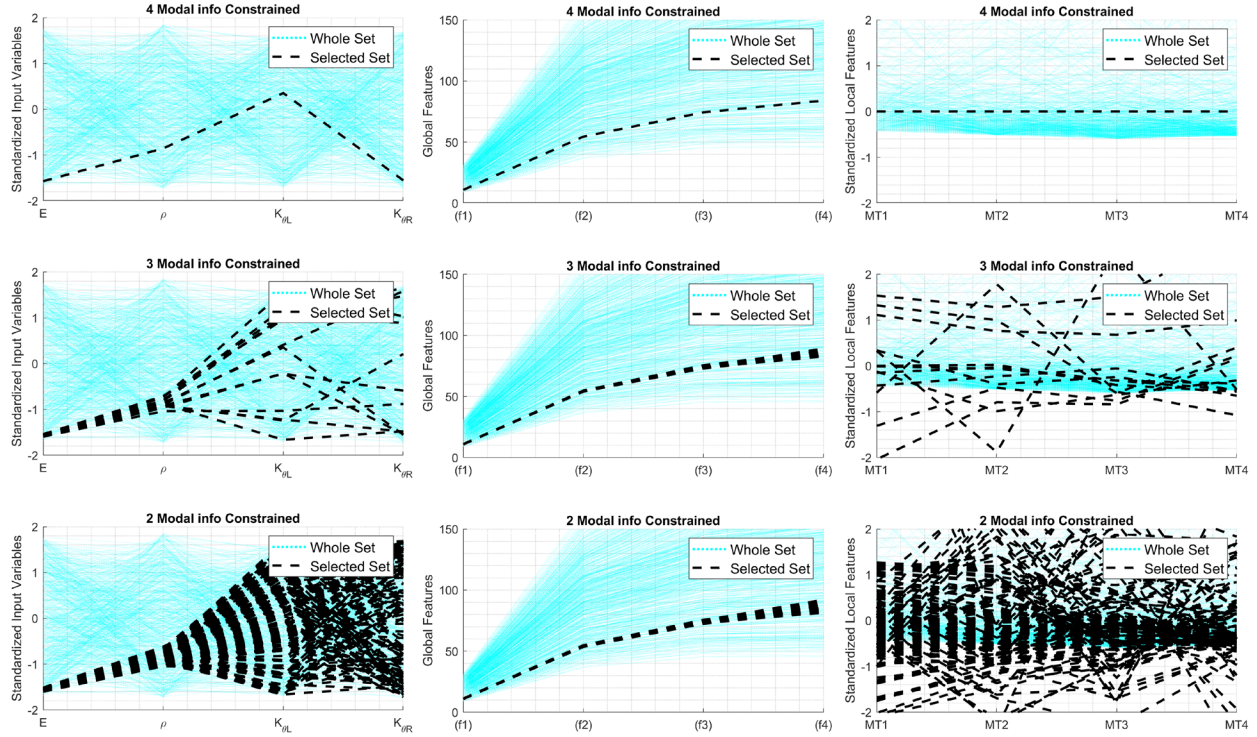


Figure 3.5 The parallel coordination plots of 4E6 samples produced by the DOE tool for the bridge model using 2, 3, and 4 modes

After determining the modal properties by various methods, it is necessary to find the system RVs (parameters) incorporated by those modal properties. The conventional structural model updating was implemented by MATLAB's "*fmincon*" function as an optimization algorithm, while the initial point was at the nominal values of RVs. The overall outcomes of the optimum values, as well as their genuine values, evaluated by laboratory measurements, are presented in table 3.1.

Table 3.1 Bridge parameters estimated by proposed SMU.

<i>Proposed Method</i>	<i>Genuine Values</i>
2284(-0.67%)	2300 M. Pa
1203(+0.27%)	1200 Kg/m³
8425(-0.88%)	8500 N.m/rad
6.34	0.00 N.m/rad

Chapter 4 Discussion and Conclusion

This work presents a new approach to the model updating of a highway bridge that determines a unique solution to the bridge's parameters and its boundary coefficients using a minimum number of measurement points. A new approach to assigning objective weights to each term in a multi-objective function is proposed. The proposed method developed an approach to identifying adequate locations for sensor placement. In comparison with existing methods in the literature, the high-order polynomial RSs are suggested to measure valid degrees of nonlinearity between system parameters and system features. Those nonlinearities are the reason that most updating methodologies are unable to find genuine optimum points.

There are still several situations in which the proposed methodology could struggle to find optimum points between the myriad set. The RSs might not attain acceptable accuracy to estimate genuine structural responses ($R^2 \leq 0.99$), or the myriad set might not be vast enough to contain genuine optimum points. In addition, the FR might be not broad enough, and the system parameters might fall inside the pre-defined FR. Most importantly, the experimentally extracted features may be fallacious, which can occur due to environmental setup and/or measurement error. Selecting the locations of the critical node can be a challenging task, especially when a large number of mode shapes are needed. Also, these locations can be impractical for experimental implementation. Nevertheless, the numerical experimentations have shown that neighboring nodes are effective as long as they are away from the stationary nodal points of Nm modes.

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